

Dynamical Symmetry Breaking in Einstein Universe

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Abstract

We investigate four-fermion interactions with N -component fermion in Einstein universe for arbitrary space-time dimensions ($2 \leq D < 4$). It is found that the effective potential for composite operator $\bar{\psi}\psi$ is calculable in the leading order of the $1/N$ expansion. The resulting effective potential is analyzed by varying the curvature of the space-time and is found to exhibit the symmetry restoration through the second-order phase transition. The critical curvature at which the dynamical fermion mass disappears is analytically calculated.

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It is the standard scenario of the cosmology to assume that in the stage of the very early universe the grand unified theory (GUT) branched off into the quantum chromodynamics and electroweak theory through the symmetry breaking caused by the Higgs mechanism. At this stage the quantum effect of the gravity is considered to be unimportant although the curvature effect due to the strong gravity still remains to be of major importance. It is thus of interest to deal with quantum field theory in curved space-time in the era of the grand unified theory. If we take the view of the dynamical symmetry breaking [1] such as the technicolor model [2], the Higgs particle playing an important role in the symmetry breaking in the GUT era is thought of as a composite system of the fundamental fermion and anti-fermion. Hence it is very interesting to discuss quantum field theory with the composite Higgs field in curved space-time [3, 4, 5]. In the present paper we deal with the four-fermion interaction theory as one of the prototypes of the composite Higgs theory. In order to get an insight into the dynamical symmetry breaking through the composite Higgs mechanism we study the phase structure of the four-fermion theory in curved space-time.

The four-fermion interaction in curved space-time is characterized by the action

$$S = \int d^D x \sqrt{-g} \left[- \sum_{k=1}^N \bar{\psi}_k \gamma^\mu \nabla_\mu \psi_k + \frac{\lambda_0}{2N} \left(\sum_{k=1}^N \bar{\psi}_k \psi_k \right)^2 \right], \quad (1)$$

where index k represents the flavor of the fermion, N is the number of the fermion species, g the determinant of the metric $g^{\mu\nu}$, γ^μ the Dirac matrix in curved space-time, ∇_μ the covariant derivative for fermion field ψ , and D the dimension of the space-time. We work in the space-time of dimension D for $2 \leq D < 4$. In the following for simplicity we neglect the summation symbol on indices of fermion field ψ . The action of Eq.(1) represents a discrete chiral symmetry in even dimensions. It is well-known that in the flat space-time (Minkowski space-time) the symmetry is broken spontaneously and the fermion acquires a dynamical mass [6, 7].

By introducing auxiliary field σ we derive action S' equivalent to action S in Eq.(1):

$$S' = \int d^D x \sqrt{-g} \left[-\bar{\psi} \gamma^\mu (\nabla_\mu + \sigma) \psi - \frac{N}{2\lambda_0} \sigma^2 \right]. \quad (2)$$

We perform the path integration over ψ and $\bar{\psi}$ in the expression for the generating functional and then find that in the leading order of the $1/N$ expansion the effective action of this model is given by

$$\Gamma[\sigma] = \int d^D x \sqrt{-g} \left(-\frac{\sigma^2}{2\lambda_0} \right) - i \text{Tr} \text{Ln}[-\sqrt{-g}(\gamma^\mu \nabla_\mu + \sigma)] + \mathcal{O}\left(\frac{1}{N}\right). \quad (3)$$

From Eq.(3) we obtain effective potential in the leading order of the $1/N$ expansion:

$$V(\sigma) = \frac{1}{2\lambda_0} \sigma^2 + \frac{1}{\int d^D x \sqrt{-g}} \text{Tr} \int_0^\sigma ds \sqrt{-g} S_F(x, y; s), \quad (4)$$

where σ is independent of the space-time coordinates and the potential $V(\sigma)$ is normalized so that $V(0) = 0$ with $S_F(x, y; s)$ defined by

$$S_F(x, y; s) \equiv \langle x | \{-i\sqrt{-g}(\gamma^\mu \nabla_\mu + s)\}^{-1} | y \rangle. \quad (5)$$

Through the definition (5) we observe that the two-point function $S_F(x, y; s)$ satisfies the following equation,

$$(\gamma^\mu \nabla_\mu + s) S_F(x, y; s) = \frac{i}{\sqrt{-g}} \delta^D(x, y), \quad (6)$$

where $\delta^D(x, y)$ is Dirac's delta function in curved space-time. According to Eq.(6) we may identify $S_F(x, y; s)$ to the propagator of the free massive fermion of mass s in curved space-time.

It is important to note here that the problem of calculating the effective potential for composite operator σ in the four-fermion interaction model in the leading order of the $1/N$ expansion reduces to that of finding the propagator of the massive free fermion in curved space-time. Fortunately in Einstein universe the two-point function $S_F(x, y; s)$ has already been calculated in Ref.[8]. The resulting expression reads

$$\begin{aligned}
S_F(x, y; s) &= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{i\omega(y^0 - x^0)} \frac{iK^{\frac{D-3}{2}}}{(4\pi)^{\frac{D-1}{2}}} \frac{\Gamma(\frac{D-1}{2} + i\beta)\Gamma(\frac{D-1}{2} - i\beta)}{\Gamma(\frac{D+1}{2})} \\
&\times \left[(s + i\gamma_0\omega)U(\mathbf{x}, \mathbf{y}) \cos\left(\frac{\sqrt{K}}{2}\sigma\right) {}_2F_1\left(\frac{D-1}{2} + i\beta, \frac{D-1}{2} - i\beta; \frac{D+1}{2}; \cos^2\frac{\sqrt{K}}{2}\sigma\right) \right. \\
&\left. + \gamma^i n_i U(\mathbf{x}, \mathbf{y}) \frac{D-2}{2} \sin\left(\frac{\sqrt{K}}{2}\sigma\right) {}_2F_1\left(\frac{D-1}{2} + i\beta, \frac{D-1}{2} - i\beta; \frac{D-1}{2}; \cos^2\frac{\sqrt{K}}{2}\sigma\right) \right], \tag{7}
\end{aligned}$$

where K and β are defined by

$$\beta \equiv \sqrt{\frac{s^2 - \omega^2}{K}}, \tag{8}$$

$$\text{and } R \equiv (D-1)(D-2)K, \tag{9}$$

respectively where R is the scalar curvature. According to Eq.(7) we find that the trace of the two-point function $S_F(x, y; s)$ is given by

$$\begin{aligned}
\text{Tr}\sqrt{-g}S_F(x, y; s) &= \int d^D x \sqrt{-g} \text{tr} S_F(x, x; s) \\
&= \int d^D x \sqrt{-g} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{isK^{\frac{D-3}{2}}}{(4\pi)^{\frac{D-1}{2}}} \left| \frac{\Gamma(\frac{D-1}{2} + i\beta)}{\Gamma(1 + i\beta)} \right|^2 \Gamma\left(-\frac{D-3}{2}\right) \text{tr}\mathbf{1}, \tag{10}
\end{aligned}$$

where $\text{tr}\mathbf{1}$ is the trace of the unit Dirac matrix. Inserting Eq.(10) into Eq.(4) we obtain the effective potential in Einstein universe:

$$V(\sigma) = \frac{\sigma^2}{2\lambda_0} + \int_0^\sigma ds \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{isK^{\frac{D-3}{2}}}{(4\pi)^{\frac{D-1}{2}}} \left| \frac{\Gamma(\frac{D-1}{2} + i\beta)}{\Gamma(1 + i\beta)} \right|^2 \Gamma\left(-\frac{D-3}{2}\right) \text{tr}\mathbf{1}. \tag{11}$$

In the second term of the right-hand side of Eq.(11), the path of the ω integral may be deformed by the Wick rotation so that one can keep the path away from the poles appearing in the Gamma functions. By changing variable ω to ω' through $\omega = i\omega'$ we rewrite Eq.(11) such that

$$V(\sigma) = \frac{\sigma^2}{2\lambda_0} - \int_0^\sigma ds \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{sK^{\frac{D-3}{2}}}{(4\pi)^{\frac{D-1}{2}}} \left| \frac{\Gamma(\frac{D-1}{2} + i\beta')}{\Gamma(1 + i\beta')} \right|^2 \Gamma\left(-\frac{D-3}{2}\right) \text{tr}\mathbf{1}, \tag{12}$$

where β' is defined by $\beta' \equiv \sqrt{\frac{s^2 + \omega'^2}{K}}$. For simplicity we shall omit the primes in ω' and β' in the following.

The effective potential $V(\sigma)$ is in general divergent for $D = 2, 3, 4$. Since the nature of the divergence is the same as that in the flat (Minkowski) space-time, the divergence can be removed by the usual flat-space renormalization for $D < 4$. (Note that our model is renormalizable for $D = 2$ in usual sense and for $D = 3$ in the sense of the $1/N$ expansion [10]. For $D \geq 4$ the model is nonrenormalizable.) The divergence associated with the vacuum energy may be curvature-dependent. This divergence, however, is absent because of our normalization of the effective potential, $V(0) = 0$, in Eq.(12). Note that in the leading order of the $1/N$ expansion the divergence at $D = 3$ in the effective potential $V(\sigma)$ happens to be absent can be seen by carefully analyzing Eq.(4) in the neighborhood of $D = 3$.

Expanding Eq.(12) asymptotically about $K = 0$ (weak curvature expansion) we find

$$V(\sigma) = V_0(\sigma) + \frac{\text{tr} \mathbf{1}}{(4\pi)^{\frac{D}{2}}} \Gamma(2 - \frac{D}{2}) \frac{1}{12} \frac{R}{D-2} \sigma^{D-2} + \mathcal{O}(R^2), \quad (13)$$

where R is given in Eq.(9) and $V_0(\sigma)$ is the effective potential in flat space-time given by

$$V_0(\sigma) = \frac{\sigma^2}{2\lambda_0} - \int_0^\sigma ds \int_{-\infty}^\infty \frac{d\omega}{2\pi} \frac{s}{(4\pi)^{\frac{D-1}{2}}} (\omega^2 + s^2)^{\frac{D-3}{2}} \Gamma(-\frac{D-3}{2}) \text{tr} \mathbf{1}. \quad (14)$$

Though the ω integration in Eq.(14) can be performed analytically, we leave it for convenience in the following argument. (Note that the expression of the weak curvature expansion (13) for $D = 3$ and 4 is in agreement with the results obtained in Refs.[4] and [5] respectively.) We impose the renormalization condition

$$\left. \frac{\partial^2 V_0}{\partial \sigma^2} \right|_{\sigma=\sigma_0} = \frac{\sigma_0^{D-2}}{\lambda_r}, \quad (15)$$

where λ_r is the renormalized coupling constant and σ_0 is the renormalization scale. Adopting the condition (15) to the effective potential (14) we find that the renormalized

coupling constant λ_r satisfies the following relation,

$$\frac{1}{\lambda_0} = \frac{\sigma_0^{D-2}}{\lambda_r} + \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{(4\pi)^{\frac{D-1}{2}}} \left\{ \omega^2 + (D-2)\sigma_0^2 \right\} (\omega^2 + \sigma_0^2)^{\frac{D-5}{2}} \Gamma\left(-\frac{D-3}{2}\right) \text{tr} \mathbf{1}. \quad (16)$$

Inserting Eq.(16) into Eq.(14) we obtain the renormalized effective potential in the Minkowski space-time:

$$V_0(\sigma) = \frac{1}{2} \left[\frac{\sigma_0^{D-2}}{\lambda_r} + \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{(4\pi)^{\frac{D-1}{2}}} \left\{ \omega^2 + (D-2)\sigma_0^2 \right\} (\omega^2 + \sigma_0^2)^{\frac{D-5}{2}} \Gamma\left(-\frac{D-3}{2}\right) \text{tr} \mathbf{1} \right] \sigma^2 \\ - \int_0^\sigma ds \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{s}{(4\pi)^{\frac{D-1}{2}}} (\omega^2 + s^2)^{\frac{D-3}{2}} \Gamma\left(-\frac{D-3}{2}\right) \text{tr} \mathbf{1}. \quad (17)$$

The nontrivial solution of the gap equation $\left. \frac{\partial V_0}{\partial \sigma} \right|_{\sigma=m_0} = 0$ provides us with the dynamical fermion mass m_0 in the Minkowski space-time. By observing the gap equation we recognize that above the critical coupling λ_{cr} given by

$$\frac{1}{\lambda_{cr}} \equiv \frac{1-D}{(4\pi)^{\frac{D}{2}}} \Gamma\left(1 - \frac{D}{2}\right) \text{tr} \mathbf{1} \quad (18)$$

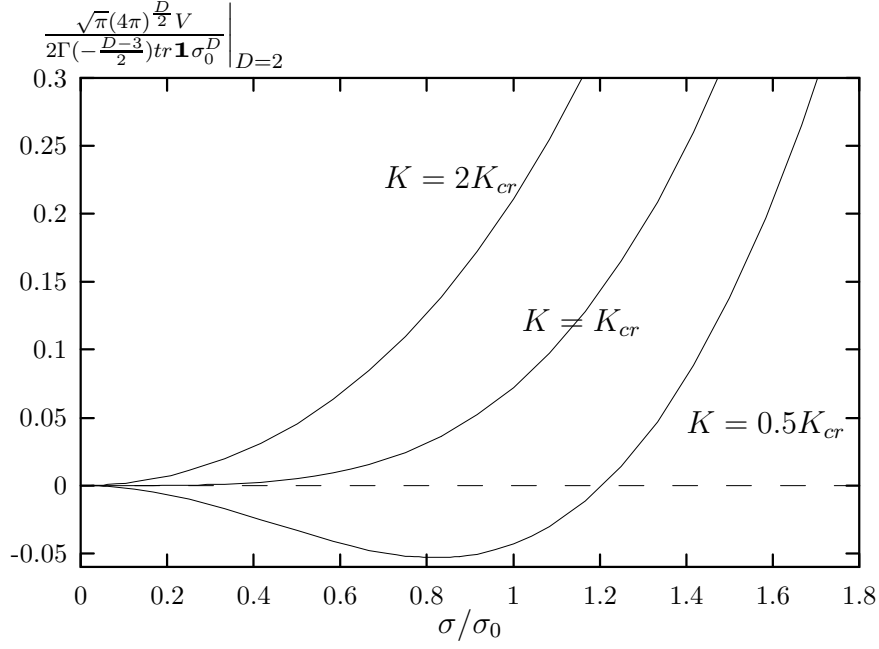
the dynamical fermion mass m_0 is generated where

$$m_0 = \left\{ \frac{(4\pi)^{\frac{D}{2}}}{\Gamma\left(1 - \frac{D}{2}\right) \text{tr} \mathbf{1}} \left(\frac{1}{\lambda_r} - \frac{1}{\lambda_{cr}} \right) \right\}^{\frac{1}{D-2}} \sigma_0 \quad \text{for } \lambda_r > \lambda_{cr}. \quad (19)$$

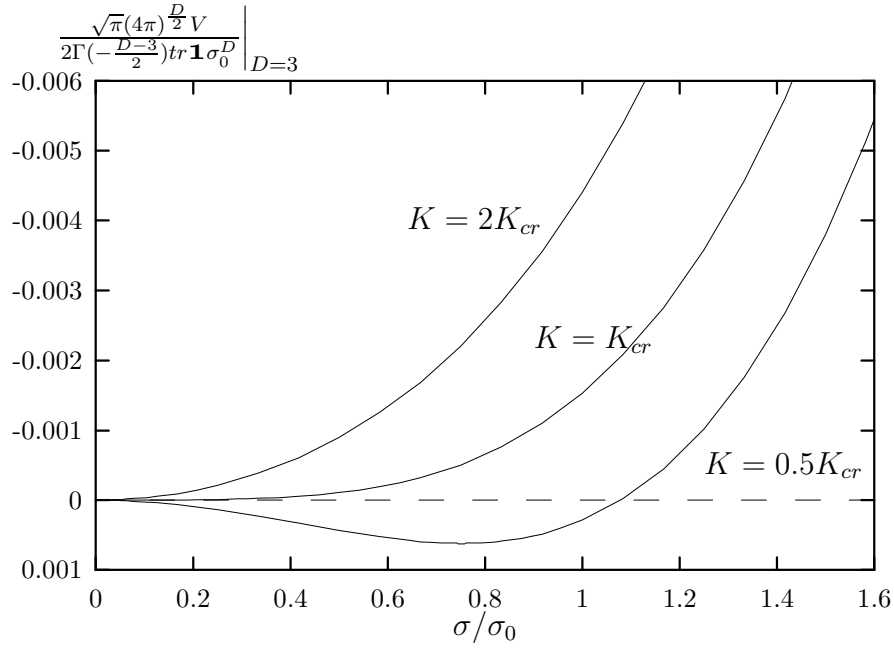
Below the critical coupling λ_{cr} the fermions remain massless: $m_0 = 0$.

Inserting Eq.(16) into Eq.(12) we obtain a renormalized expression of the effective potential in Einstein universe :

$$V(\sigma) = \frac{1}{2} \left[\frac{\sigma_0^{D-2}}{\lambda_r} + \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{(4\pi)^{\frac{D-1}{2}}} \left\{ \omega^2 + (D-2)\sigma_0^2 \right\} (\omega^2 + \sigma_0^2)^{\frac{D-5}{2}} \Gamma\left(-\frac{D-3}{2}\right) \text{tr} \mathbf{1} \right] \sigma^2 \\ - \int_0^\sigma ds \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{s K^{\frac{D-3}{2}}}{(4\pi)^{\frac{D-1}{2}}} \left| \frac{\Gamma\left(\frac{D-1}{2} + i\beta\right)}{\Gamma(1 + i\beta)} \right|^2 \Gamma\left(-\frac{D-3}{2}\right) \text{tr} \mathbf{1}. \quad (20)$$



(a) Behavior of the effective potential for $D = 2$ and $\lambda_r = 3\lambda_{cr}$



(b) Behavior of the effective potential for $D = 3$ and $\lambda_r = 3\lambda_{cr}$

Fig. 1 Behavior of the effective potential as a function of σ for some typical value of curvature K in the case that $\lambda_r > \lambda_{cr}$.

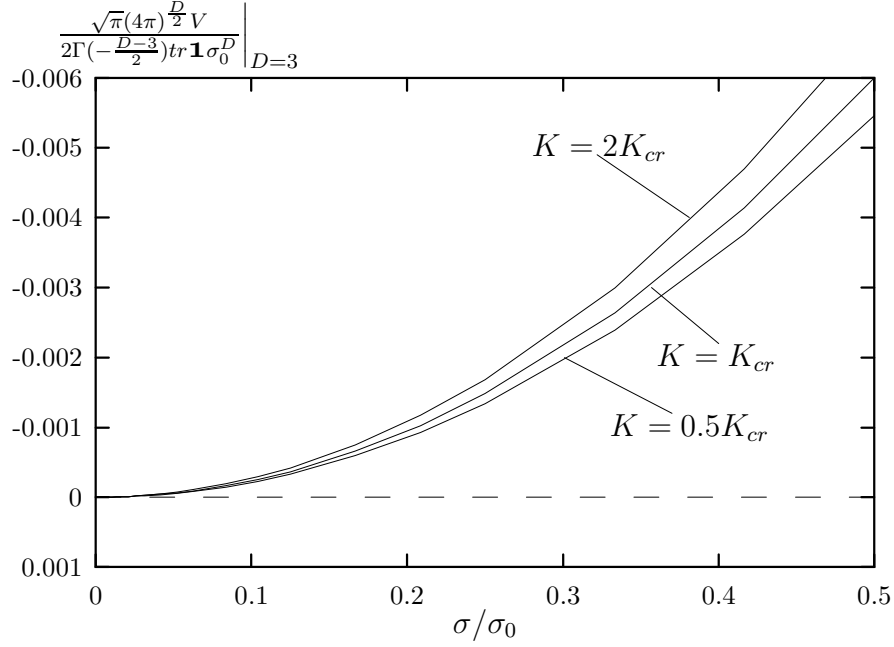


Fig. 2 Behavior of the effective potential as a function of σ for some typical value of curvature K in the case that $\lambda_r < \lambda_{cr}$ for $D = 3$.

By the use of the expression of the effective potential as given above we may study the behavior of the effective potential as a function of σ through the numerical integrations. In the numerical integration in ω we introduced a suitable upper and lower bound and performed the numerical integration between these bounds. By assuming the sufficiently large absolute value of these bounds and checking the stability of the integral under the change of the bounds we obtained the numerical value for $V(\sigma)$ for each value of σ with λ_r and K kept fixed.

In Fig.1 the behavior of the effective potential $V(\sigma)$ as a function of σ is plotted for the case $\lambda_r > \lambda_{cr}$ (as a typical example we choose $\lambda_r = 3\lambda_{cr}$) for which the chiral symmetry is broken and the fermion mass is generated in the flat space where $K = 0$. The behavior of the effective potential is shown only for $D = 2$ and 3 . We clearly observe that the behavior of the effective potential $V(\sigma)$ for $K = 0$ which is typical of the broken phase is gradually changed into the typical behavior in the symmetric phase as we increase the curvature K . The critical curvature at which the symmetry restoration takes place

can be calculated analytically as will be shown later. In Fig.2 the behavior of $V(\sigma)$ is presented for the case $\lambda_r \leq \lambda_{cr}$ (as a typical example we choose $\lambda_r = \lambda_{cr}/2$ and the space-time dimension $D = 3$). We observe that the symmetric phase stays symmetric if we change the curvature.

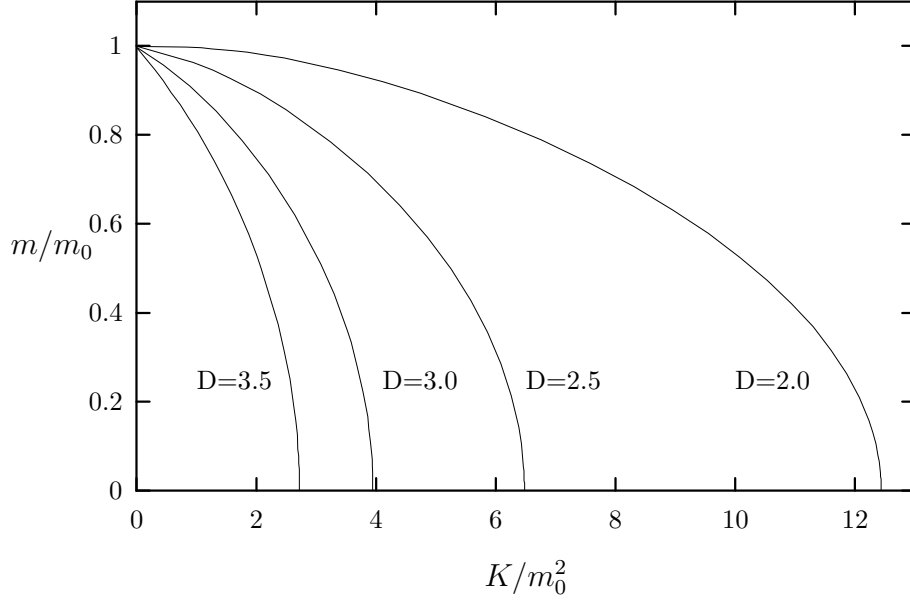


Fig. 3 Behavior of the dynamical fermion mass as a function of curvature K for $D = 2, 2.5, 3$ and 3.5 .

The dynamical fermion mass m in Einstein universe is calculated by solving the gap equation $\left. \frac{\partial V}{\partial \sigma} \right|_{\sigma=m} = 0$:

$$\begin{aligned} \frac{\sigma_0^{D-2}}{\lambda_r} + \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{(4\pi)^{\frac{D-1}{2}}} \left[\left\{ \omega^2 + (D-2)\sigma_0^2 \right\} (\omega^2 + \sigma_0^2)^{\frac{D-5}{2}} \right. \\ \left. - K^{\frac{D-3}{2}} \left| \frac{\Gamma(\frac{D-1}{2} + i\beta)}{\Gamma(1 + i\beta)} \right|^2 \right] \Gamma\left(-\frac{D-3}{2}\right) \text{tr} \mathbf{1} = 0, \end{aligned} \quad (21)$$

where $\beta = \sqrt{\frac{m^2 + \omega^2}{K}}$. Fig.3 represents the behavior of the dynamical fermion mass which is obtained by solving Eq.(21) numerically for $D = 2.0, 2.5, 3.0$ and 3.5 . The numerical integration in Eq.(21) was performed in the same way as in Fig.1 and Fig.2. As we have

observed in Fig.1 and Fig.3, the phase transition caused by the change of the curvature is of the second order. This means that the critical curvature may be calculated by taking the limit $m \rightarrow 0$ in the gap equation (21). We let $m \rightarrow 0$ in Eq.(21) and perform the integration analytically. We find

$$\frac{1}{\lambda_r} + \frac{D-1}{(4\pi)^{\frac{D}{2}}} \Gamma(1 - \frac{D}{2}) \text{tr} \mathbf{1} - \left(\frac{K_{cr}}{\sigma_0^2} \right)^{\frac{D-2}{2}} \frac{1}{\sqrt{\pi}(4\pi)^{\frac{D}{2}}} \Gamma(\frac{D-1}{2}) \Gamma(\frac{D}{2}) \Gamma(1 - \frac{D}{2}) \text{tr} \mathbf{1} = 0. \quad (22)$$

Taking into account Eq.(19) with Eq.(18) we rewrite Eq.(22) in the following form,

$$\left[m_0^{D-2} - K_{cr}^{\frac{D-2}{2}} \frac{1}{\sqrt{\pi}} \Gamma(\frac{D-1}{2}) \Gamma(\frac{D}{2}) \right] \Gamma(1 - \frac{D}{2}) = 0. \quad (23)$$

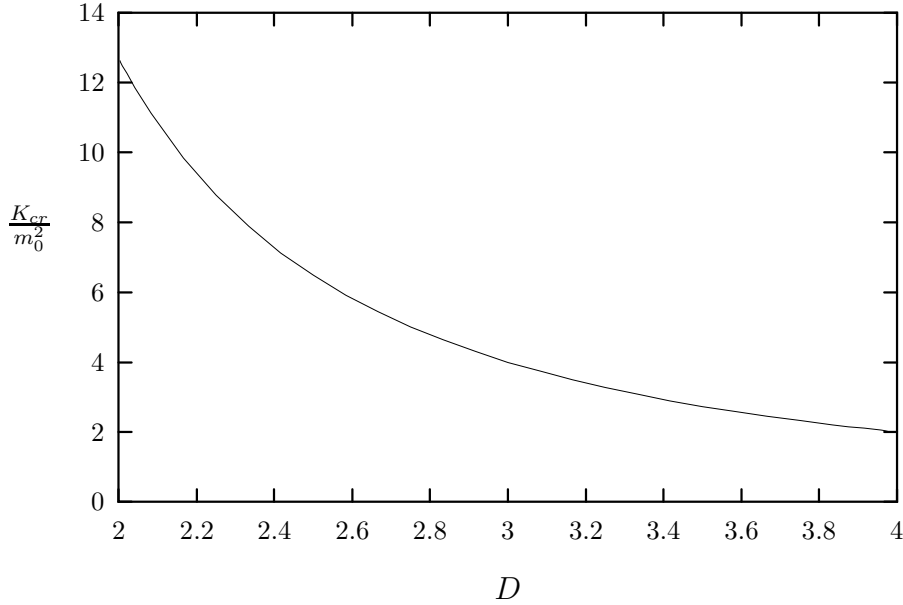


Fig. 4 Behavior of the critical curvature K_{cr} as a function of dimension D .

In Fig.4 we show dependence of the critical curvature K_{cr} to dimension D . We see

from Eq.(23) that for some special values of D K_{cr} is given by

$$\left\{ \begin{array}{ll} K_{cr} = 4e^{2\gamma_E} m_0^2 & \text{at } D = 2.0, \\ K_{cr} = \left(\frac{8}{\pi}\right)^2 m_0^2 & \text{at } D = 2.5, \\ K_{cr} = 4m_0^2 & \text{at } D = 3.0, \\ K_{cr} = \left(\frac{16}{3\sqrt{2\pi}}\right)^{\frac{4}{3}} m_0^2 & \text{at } D = 3.5, \\ K_{cr} = 2m_0^2 & \text{at } D = 4.0, \end{array} \right. \quad (24)$$

where γ_E is the Euler constant.

In Eq.(24) the critical curvature K_{cr} for $D = 2$ reproduces the result obtained in Ref.[11]. It is well-known that in the case of the two dimensional space-time the field theory in $R \otimes S$ that is regarded as an Euclidean analog of the Einstein space-time is equivalent to the finite-temperature field theory. Since the effective potential for the two-dimensional finite-temperature four-fermion theory is known [12], we compare our result (12) with the previous finite-temperature result. In the two-dimensional limit $D \rightarrow 2$ the effective potential (12) reduces to

$$V(\sigma) \rightarrow \frac{\sigma^2}{2\lambda_0} - \int_0^\sigma ds \int_{-\infty}^\infty \frac{d\omega}{3\pi} \frac{s}{\sqrt{K}} \frac{1}{\beta} \tanh \pi\beta \quad (25)$$

$$= \frac{\sigma^2}{2\lambda_0} - \frac{\sqrt{K}}{2\pi} \sum_{n=-\infty}^\infty (\sqrt{\omega_n^2 + \sigma^2} - \sqrt{\omega_n^2}), \quad (26)$$

where $\omega_n = \frac{2n+1}{2}\sqrt{K}$. Here we utilized the formula,

$$\sum_{n=1}^\infty \frac{1}{x^2 + (2n-1)^2} = \frac{\pi}{4x} \tanh \pi \frac{x}{2}. \quad (27)$$

With recourse to the following relation between the curvature K and the temperature T

$$\frac{1}{k_B T} = \frac{2\pi}{\sqrt{K}}, \quad (28)$$

with k_B the Boltzmann constant, we realize that the effective potential (12) in the Einstein universe for $D = 2$ is in agreement with that of the finite-temperature four-fermion

theory in $D = 2$ and the result for $D = 2$ given in Eq.(24) is consistent with the well-known formula of the critical temperature for the four-fermion interaction model at finite-temperature (See Ref.[12]).

We have found the presence of the curvature-induced dynamical symmetry restoration for the system of self-interacting fermions in Einstein universe in the leading order of the $1/N$ expansion. The dynamical fermion mass is found to disappear through the second-order phase transition. The critical curvature at which the symmetry restoration takes place is calculated analytically.

Since Einstein universe is the static universe, our result in the present paper can not be applied directly to the evolution of the universe in its early stage. Our model, however, gives a hint on the scenario of the symmetry breaking in the early universe. In order to make our result more realistic it is necessary for us to extend our analysis to the more general universe such as the Robertson-Walker universe and study the Einstein equation with the energy-momentum tensor calculated in the underlying theory. It is our hope to give a fundamental contribution to the understanding of the dynamical origin of the inflationary expansion of the universe.

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